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Effects of Statistical Distribution of Micro-Cracks on Scatter of Fracture Toughness of Steels under Conditions of Brittle Failure

Vliv statistické distribuce mikrotrhlin na rozptyl lomové houževnatosti ocelí za podmínek křehkého porušení

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The aim of the paper is an analysis of the effects of the statistical distribution of micro-cracks on scatter of fracture toughness of steels under conditions of brittle failure. The results are utilized for reliability assessment of selected functional parts. The reliability considered as a complementary probability of brittle fracture initiation is discussed in dependence on the character of the statistical distribution of micro-crack sizes, mechanical properties of steel, mechanisms of energy dissipation during cracks propagation, a variation of loading, stress state of the functional part and its service life. This probability approach is compared with deterministic reliability access originating from the computation of the safety factor. Its rational evaluation as a function of the acceptable probability of fracture instability provides high economic effects by saving materials and energy.

***Key words**: microcracks; HRR stress field; brittle fracture; stress intensity factor; fracture probability; reliability; structural component*

Předkládaný článek analyzuje účinky statistické distribuce mikrotrhlin na rozptyl lomové houževnatosti ocelí za podmínek křehkého porušení. Výsledky jsou využity pro hodnocení spolehlivosti vybraných funkčních částí – strojních dílů. Spolehlivost je v článku uvažována jako komplementární pravděpodobnost iniciace křehkého porušení. Je posuzována v závislosti na charakteru statistické distribuce velikosti mikrotrhlin, mechanických vlastností oceli, mechanismech disipace energie při šíření trhlin, změně zatížení, stavu napětí funkční části a její životnosti. Tento pravděpodobnostní přístup je porovnán deterministickým spolehlivostním přístupem, který vychází z výpočtu koeficientu bezpečnosti. Jeho racionální hodnocení jako funkce přijatelné pravděpodobnosti nestability lomu poskytuje vysoké ekonomické efekty, které šetří materiál a energii. Článek pojednává o vývoji statistického modelu pro predikci pravděpodobnosti křehkého porušení. Vzhledem k tomu, že kritickým okamžikem je iniciace mikrotrhlin, byla základní pravděpodobnost křehkého porušení kvantifikována jako funkce rozdělení velikosti a tvaru mikrotrhlin, jejich orientace, elastických a plastických charakteristik materiálu. Ze statistické teorie nejslabšího článku, byla vypočtena celková pravděpodobnost křehkého porušení pro homogenně i nehomogenně namáhanou část konstrukce. Vypočtená celková pravděpodobnost křehkého porušení může být využita nejen pro mikrostrukturní design křehkých materiálů, ale také pro novou formulaci pravděpodobnostního návrhu komponent konstrukčních částí.

***Klíčová slova**: mikrotrhliny; HRR napěťové pole; křehký lom; faktor intenzity napětí; pravděpodobnost lomu; spolehlivost; konstrukční prvky*

Structural components made from brittle materials, such as ceramics, intermetallics, glasses or carbon steels at low temperatures must be designed regarding flaws, holes, and inclusions in structure. The local stress concentrations around these mentioned defects are followed by micro-cracking. If these micro-cracks extend and interact with each other so that they grow in an unstable manner, then a macroscopic failure may arise. The usual combination of high strength and low fracture toughness of brittle materials leads to relatively small critical crack size the detection of which by current non-destructive evaluation methods is very difficult. As a result, service reliability of components made from brittle materials is very sensitive to microstructural parameters.

Low-temperature transgranular cleavage of carbon structural steels has been experimentally proved to be initiated by micro-cracking of carbides [1 – 3]. Local heterogeneity in deformation may result in the initiation of micro-cracks and their propagation into the matrix whenever the applied stress σ exceeds the local cleavage strength σ_f [4]:

$$\sigma \geq \sigma_f = (\beta / 2)^{1/2} \frac{k_{Ia}}{\sqrt{d_p}}, \qquad (1)$$

where $k_{Ia} = [2E\gamma_{eff} / (1-\nu^2)]^{1/2}$ is micro-crack arrest toughness introduced by Hahn [4], d_p is the micro-crack size, β is a micro-crack shape factor; $\beta = \pi$ for penny-shaped and β = 4/π for trough thickness micro-crack [1 – 7], E is Young's modulus, ν is Poisson ratio and γ_{eff} is the effective surface energy.

Experimental investigations of low-temperature brittle fracture in steels was over the past years complemented by attempts to model the fracture process by statistical methods [2, 5 – 12], using local criteria for the initiation of micro-cracks. These approaches can reveal the relationship between the microstructural parameters and macroscopic mechanical properties. From size distribution of carbides using the Weibull's weakest link statistical theory the integral probability of cleavage failure and the temperature dependence of scattering in fracture toughness was computed [5, 8 – 13].

Some parameters affecting the probability of fracture as the character of the stress field, shape and orientation of micro-cracks or the volume where the micro-cracking process is activated have not yet been discussed in detail. This paper is concerned with the probability of brittle fracture in steels loaded under conditions of non-homogenous elastic and elastic-plastic stress field, and it provides a method for calculating the effect of these conditions on the fracture instability.

1. Probability of micro-cracking

The initiated micro-crack obeying criterion given in Eq. (1) propagates further more easily if the cleavage planes in the matrix of the steel are favorably orientated relative to the acting stress. When substantial misalignment between these planes exists, or when micro-cracks are too small to satisfy the propagation criterion, it causes stable micro-cracks formation. Similarly, the deviation between applied stress direction and the stress perpendicular to the cleavage plane α makes micro-crack propagation into matrix difficult and the local cleavage strength σ_f given by Eq. (1) is by $1/\cos^2\alpha$ times higher [14]. Then for every magnitude of local stress σ there is a certain critical size of micro-crack, at which they can spread into the steel matrix:

$$d_{pf}(\sigma) = \frac{\beta k_{Ia}^2}{2\sigma^2 \cos^4 \alpha}. \qquad (2)$$

The probability of such an event is given by:

$$p_f(\sigma) = \Pr(d_p \geq d_{pf}(\sigma)) = \int_{dpf(\sigma)}^{\infty} \int_0^{\pi/2} \xi(\alpha)\psi(d_p)\mathrm{d}_\alpha \mathrm{d}_{dp}, \qquad (3)$$

where:

$$\psi(d_p) = \delta_0 d_{p0}^{-\delta_0} d_p^{(\delta_0 - 1)} \exp[-(d_p / d_{p0})^{\delta_0}] \qquad (4)$$

is the common Weibull's probability density function of micro-cracks with size d_{p0} and shape δ_0 parameters,

$$\xi(\alpha) \cong A_\alpha \sin\alpha \qquad (5)$$

is the probability density of misalignment or disorientation angle α (Fig. 1) received from uniform projection of perpendiculars to the cleavage plane [14] and A_α is a constant.

The elementary probability that at least one carbide within isostressed volume element $\delta V(\sigma)$, e.g., ahead of macrocrack tip (Fig. 1) can be investigated from the sum:

$$\delta P_f(\sigma) = 1 - \sum_{i=0}^{\infty} \zeta(\delta V, i)[1 - p_f(\sigma)]^i = 1 - \exp[-N_V \delta V p_f(\sigma)], \qquad (6)$$

where $\zeta(\delta V, i) = 1/i!(N_V\delta V)^i \cdot \exp(-N_V \delta V)$ is the probability that i microcracks initiate in the volume element $\delta V(\sigma)$ corresponding to Poisson´s distribution and $[1 - p_f(\sigma)]^i$ is a complementary probability that i micro-cracks will be initiated into the considered volume element. For homogenously stressed volume V the final brittle fracture probability P_f is easy to calculate from Eq. (6) replacing δV by V and $P_f = P_f(\sigma)$.

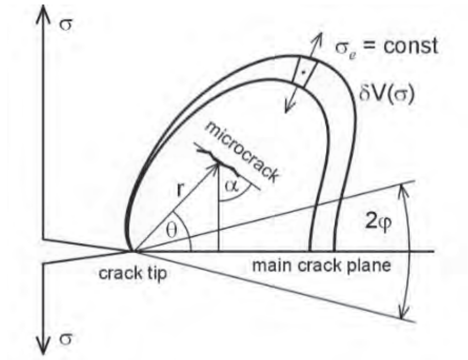

Fig. 1 Schematic illustration of isostressed volume element and active wedge zone ahead of macrocrack tip

Obr. 1 Schematická ilustrace namáhaného objemového prvku a klínová aktivní zóna na čele makrotrhliny

Nonhomogenous stress field around the sharp macro-crack tip at small scale yielding conditions satisfies the HRR singular solution [15]:

$$\sigma_{ij}(r,\theta) = \sigma_0 \left[\left(\frac{1-\nu^2}{I_n}\right)\left(\frac{K_I}{\sigma_0\sqrt{r}}\right)^2\right]^{1/(n+1)} \tilde{\sigma}_{ij}(n,\theta), \qquad i,j = r,\theta \qquad (7)$$

where σ_0 is the yield stress, n is the work hardening exponent following from the constitutive law of $\varepsilon/\varepsilon_0 = \alpha_0(\sigma/\sigma_0)^n$, ε_0 is the yield strain, α_0 is a material constant of order of units, I_n is dimensionless parameter weakly dependent upon the work hardening exponent n, $\tilde{\sigma}_{ij}(n,\theta)$ are angular functions of n and K_I is Mode I tress intensity factor. At low temperatures, because the plasticity is small, stress field around macro-crack tip approximates the linear elastic asymptotic solution by Williams [16]:

$$\sigma_{ij}(r,\theta) = \frac{K_I}{\sqrt{2\pi r}} h_{ij}(\theta), \qquad i,j = r,\theta \qquad (8)$$

where $h_{ij}(\theta)$ a is a dimensionless function of θ. Effective stress field around crack tip $\sigma_e(r,\theta)$ has been considered as the maximum eigenvalue calculated from HRR and elastic stress tensors given by Eqs. (7) and (8).

Isostressed volume element is easily an integral of $\sigma_e(r,\theta)$ in the following form (Fig. 1):

$$\delta V(\sigma_e) = 2b\int_0^{\varphi} r\delta r \,\mathrm{d}\,\theta, \qquad (9)$$

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where φ is an angle of the active wedge region ahead of the macro-crack tip and b is the characteristic width of the crack front [9]. The total probability of brittle fracture initiation can now be established by integrating Eq. (6) within the limits of the lowest $\sigma_{f\min}$ and the highest $\sigma_{f\max}$ local strengths. These extreme values of the local cleavage strength were calculated using Eq. (1) from the largest $d_{p\max}$ and smallest $d_{p\min}$ micro-cracks given by the statistical distribution of them (Eq. (4)). Integrated Eq. (6) within the active region in nonhomogenous stress field around the crack tip makes it possible to calculate the total fracture probability P_f as a function of the stress intensity factor K_I. At the fracture instability, $K_I = K_{Ic}$ represents $100P_f$% quantile of the experimentally assessed statistical distribution of fracture toughness.

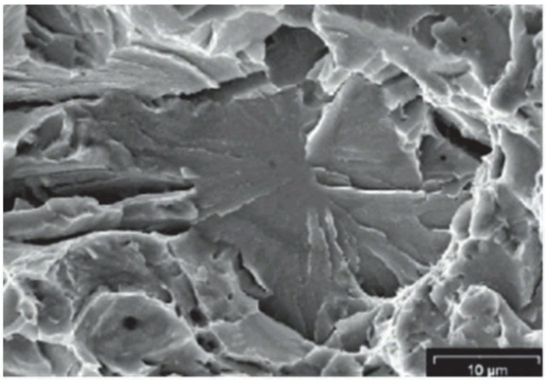

Fig. 2 Star shaped cleavage facet initiated by local stress concentration in its center, Ni-Cr steel tested at the temperature of 143 K

Obr. 2 Hvězdicovitá štěpná fazeta iniciovaná lokálním namáháním uprostřed, Ni-Cr ocel testována při teplotě 143 K

The relative frequencies of face sizes were subjected to the statistical processing of the least squares method, and the parameters of the Weibull's statistical distribution given by Eq. (4) have been found to be, d_{p0} = 26.3 μm and δ_0 = 2.28. A statistical coincidence test confirmed good agreement obtained between the analytically determined shape of the probability density and the experimentally ascertained distribution.

The entire testing procedure of Ni-Cr steel and its results are published elsewhere [12]. The statistical distribution of micro-crack sizes given by Eq. (4) was employed for numerical calculation of the total fracture probability P_f as a function of homogenous acting stress σ. Calculated curves of P_f for homogenously stressed volumes m/N_V in average containing $m = 1$, 10, 10^2, 10^3, and 10^6 micro-cracks are given in Fig. 3. Increasing volume m/N_V lowers the strength σ_f of the body and the curve is getting straight so that the transition from $P_f = 0$ to $P_f = 1$ state is jumped. The influence of the micro-crack shape factor β and the way of space array on the total fracture probability P_f is illustrated in Fig. 4. The probability P_f is calculated according to Eq. (6) for the volume corresponding to one cracked carbide $V = 1/N_V$, then $P_f = 1 = \exp[-p_f(\sigma)]$.

Comparison of $P_f(\sigma)$ curves for elastic and HRR stress field at 113 K clearly demonstrates the affirmative effect of small-scale yielding on the growth of the fracture toughness. Fig. 5 illustrates how the probability of fracture P_f depends on the stress field singularity around a sharp crack tip. The experimentally assessed statistical distribution of fracture toughness K_{Ic} of the investigated steel at temperatures of 113 K of fracture toughness K_{Ic} of the investigated steel at temperatures 113 K and 143 K are predicted using integral form of Eq. (6) for HRR Eq. (7) or elastic Eq. (8) stress field. The following Fig. 6 shows the influence of changes in micro-cracks shape expressed by parameter β, yield stress σ_0 and effective surface energy γ_{eff} on the total fracture probability P_f as a function of fracture toughness considering HRR stress field action at the temperature of 143 K. It is obvious from the curves that lowering yield stress σ_0 and increasing effective surface energy γ_{eff} suppress the brittle fracture.

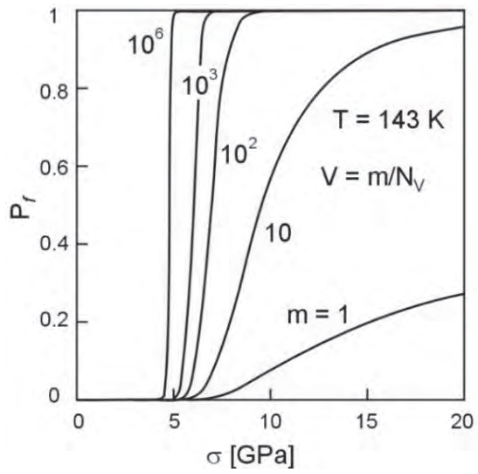

Fig. 3 Dependence of the total probability of brittle fracture on local stress in volumes corresponding to various number m of microcracks in Ni-Cr steel

Obr. 3 Závislost celkové pravděpodobnosti křehkého lomu na lokálním napětí v objemu odpovídajícímu různému počtu m mikrotrhlin v oceli Ni-Cr

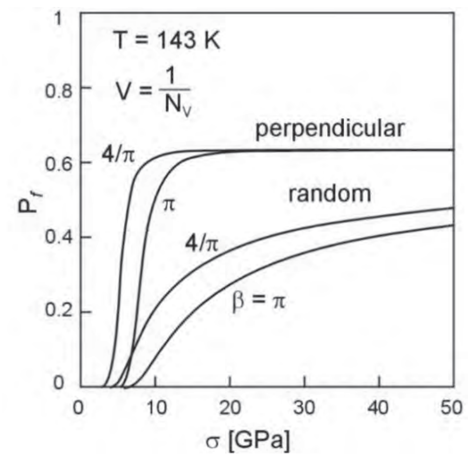

Fig. 4 Dependence of brittle fracture on local stress for perpendicular and random orientation of cleavage planes with respect to local acting stress, also two various shapes of microcracks, (penny-shaped $\beta = \pi$ and through the thickness $\beta = 4/\pi$) are considered

Obr. 4 Závislost křehkého porušení na lokálním napětí pro kolmou a náhodnou orientaci štěpných rovin s ohledem na lokálně působící napětí, jsou také zvažovány dva různé tvary mikrotrhlin (kruhový tvar $\beta = \pi$ a přes tloušťku $\beta = 4/\pi$)

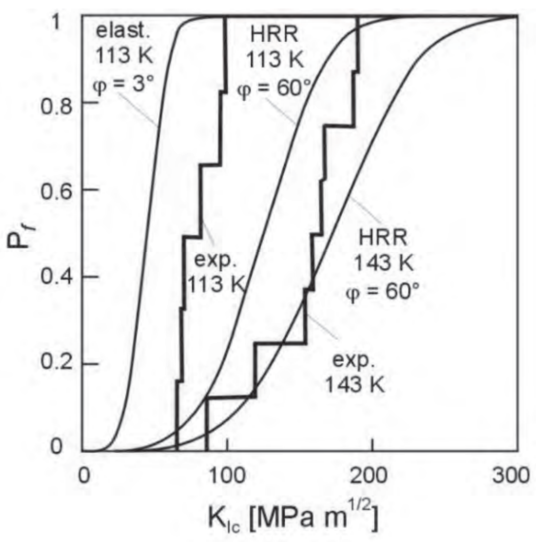

Fig. 5 Total probability of fracture predicted as a function of fracture toughness for two different temperatures considering elastic and small-scale yielding HRR stress field

Obr. 5 Celková pravděpodobnost porušení predikována jako funkce lomové houževnatosti pro dvě různé teploty s ohledem na pružnost a malou plasticitu, HRR napěťového pole

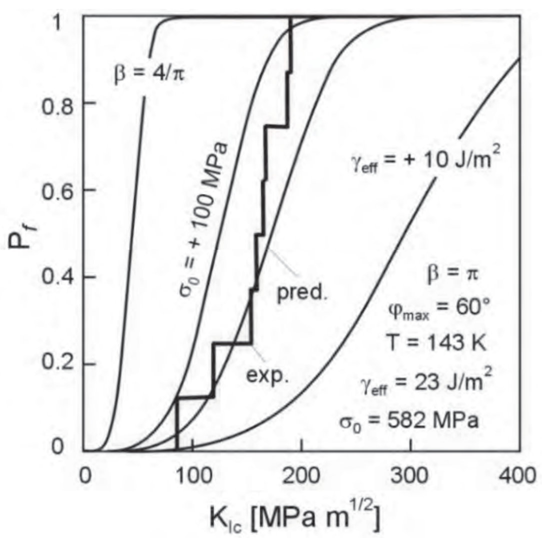

Fig. 6 Effect of increase in temperature, effective surface energy and change of micro-crack shape on the predicted statistical distribution of fracture toughness of testing steel at a temperature of 143 K

Obr. 6 Efekt zvýšení teploty, efektivní povrchové energie a změna tvaru mikrotrhliny na předpokládanou statistickou distribuci lomové houževnatosti zkoušené oceli při teplotě 143 K

2. Results and Application

The model of brittle fracture was applied to experimental results of Ni-Cr steel [12]. The steel was heat treated to give a structure of tempered martensite and roughly spherical carbides. Transmission electron microscopy at a magnification of 13,500 was employed to investigate the distribution of carbides and their area density. True stress-strain curves, yield stress σ_0 and strain hardening exponent n were assessed from uniaxial tensile tests at low temperatures where brittle fracture prevails.

Plane strain fracture toughness, K_{Ic} was evaluated in the lower bound temperature range on SENB specimens 25 mm thick according to ASTM E 399. Fractography investigation using SEM indicated that over the tested temperature range from 93 K to 153 K the fracture surface was mainly created by cleavage facets initiated at larger carbides. The brittle fracture zone of the fracture surface beyond the ductile zone consisted of cleavage facets. Carbides at fracture surfaces of the investigated steel were identified to act as subsidiary sources of cleavage facets formation. Frequent star-shaped cleavage facets at fracture surfaces of broken specimens were evidently initiated by local stress concentration in their centers where carbides triggering the facets were recognized (Fig. 2). The direction of river markings from the center of a star-shaped facet to its periphery clearly shows that observed micromechanism of the initiation of micro-cracks is controlled by local stress in the area around carbides.

3. Discussion

Micro-cracking is connected with main crack propagation in steels under condition of the brittle failure, and it considerably affects the mechanical behavior of engineering parts. Clarifying the effect of micro-cracking, and interactions of micro-cracks in the field of the macro-crack tip is the key to a clear understanding of cracking behaviors of brittle materials generally. Sometimes, a too complicated description of arrays of microcracks demands a simplified statistical treatment of this problem.

The proposed solution applied to Ni-Cr steel at very low temperatures evinces how micro-cracking originated in a bear influences the fracture instability of the main crack in a body. Not only the size of the nucleated micro-cracks but also the way, in which the orientation of cleavage planes in the matrix affects the total probability of fracture, is taken into account. It is shown in Eq. (3), that this factor can substantially increase the critical size of micro-crack, and then randomly orientated micro-cracks diminish the probability of the brittle fracture. This effect of the micro-cracks orientation is stronger in homogenously loaded body than in the nonhomogenous stress field, such as one around the macro-crack tip. Increasing effective surface energy, lowering yield stress and localized plasticity in the nearness of the macro-crack tip reduce the risk of creation of the brittle fracture. The solution based on the stress singularity described by the stress intensity factor makes it possible to predict the statistical distribution of fracture toughness. Elastic stress field singularity ahead of crack tip promotes much more brittle fracture then small-scale yielding HRR stress field, and it diminishes fracture toughness. Through thickness micro-cracks when $\beta = 4/\pi$ are more sensitive to create brittle fracture instability than penny-shaped micro-cracks nucleated with $\beta = \pi$. All these general conclusions can be directly utilized in the microstructural design of steels operating at low temperatures or in conditions evoking embrittlement.

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As it has been analyzed earlier, the actual strength or fracture toughness of the material could vary depending on microstructural parameters. In addition, it is usually difficult to precisely predict the external loads acting on the component made from the material under actual service conditions. The risk of the brittle fracture can be expressed in terms of statistical distributions of local maximum effective stress $\sigma_{e\,\max} = \sigma$ of the brittle fracture can be $\varphi_1(\sigma_1)$ and local cleavage strength $\varphi_2(\sigma_2)$ as follows [17]:

$$P_f = \Pr(\sigma \geq \sigma_f) = \int_0^{+\infty} \varphi_1(\sigma) \int_0^{\sigma} \varphi_2(\sigma_f)\, \mathrm{d}\sigma_f\, \mathrm{d}\sigma. \quad (10)$$

On the other hand, the safety factor locally defined as, $k_p = \sigma_f / \sigma_{e\,\max}$, behaves due to the variability of σ_f and $\sigma_{e\,\max}$ randomly with statistical probability density:

$$K(k_p) = \int_0^{+\infty} \sigma \varphi_1(\sigma) \varphi_2(k_p \sigma)\, \mathrm{d}\sigma \quad (11)$$

and Pr ($k_p \leq 1$) equals to the probability P_f. This relationship is an attempt to span differences between deterministic and probabilistic approaches in the design of engineering components. Furthermore, utilizing Eq. (6) in the integral form, the microstructural and macroscopical factors affecting failure of the component can be linked together to acquire more precise estimation of its reliability. Fundamentally, designing aimed at prevention of service failures of components made from brittle material is a statistical problem including its defects array and loading of components.

Conclusions

A statistical model for prediction of brittle fracture probability has been developed. Considering that the critical event is micro-crack initiation, its elementary probability has been quantified as a function of size and shape distributions of micro-cracks, theirs orientation, elastic and plastic characteristics of the material. From the weakest link statistical theory, the total brittle fracture probability has been computed for homogenously and nonhomogenously loaded part of an engineering component. Applying the model to nonhomogenous stress field described by KI—singularity around sharp crack tip, the statistical distribution of fracture toughness has been predicted. Increasing effective surface energy, lowering yield stress and localized plasticity ahead of the macrocrack tip reduce the brittle fracture probability and increase brittle fracture toughness. The model has been proved on Ni-Cr steel having a microstructure of tempered martensite and precipitated carbides acting as micro-cracks initiation origins. Calculated total probability of brittle fracture can be exploited not only for the microstructural design of brittle materials but also for new formulation in the probabilistic design of components.

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